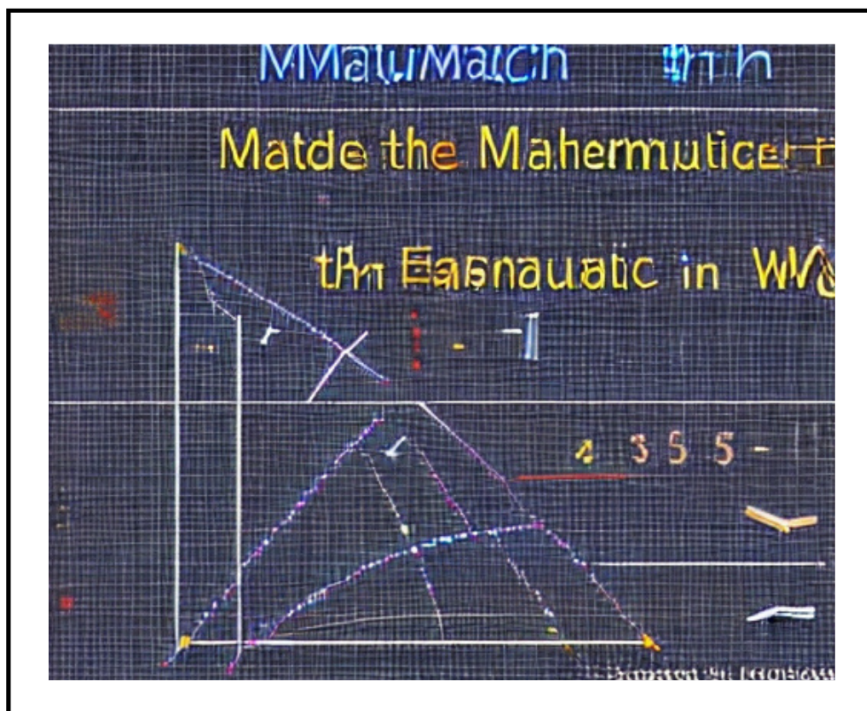


MATHEMATICS IN THE MODERN WORLD



HGBaquiran College, Inc.

PRESENTED TO

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A PROJECT BY

BSIT/B.Sc. IT 1st Year (BLOCK-B)

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Lesson 1

Mathematics plays a crucial role in the modern world, underpinning much of the technology and scientific research that shapes our society. It is used in fields such as finance, engineering, physics, computer science, and many others. For example, mathematical algorithms are used in search engines, GPS navigation systems, and image recognition software. In finance, mathematical models are used to evaluate investment opportunities and measure risk. In physics, mathematical equations are used to describe the behavior of the natural world. Additionally, mathematics is also used in many areas of everyday life, such as cooking, shopping, and playing games.

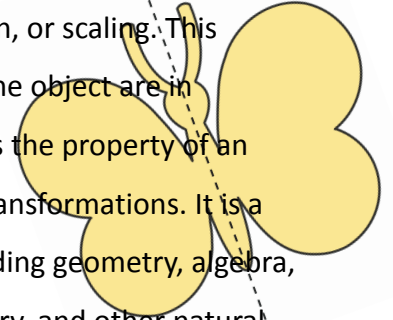
In this chapter, you will learn to identify patterns in nature and regularities in the world, understand the importance of Mathematics in your daily life, articulate the nature of mathematics, its expression, representation, and usage and discuss the role of mathematics in various disciplines. By the end of this chapter, you should appreciate the significance of mathematics as a human endeavor.

PATTERNS IN NATURE AND REGULARITIES IN THE WORLD

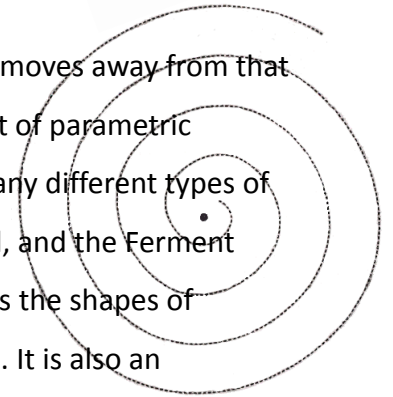
Number patterns and counting are closely related, as counting occurs when there is a pattern. In turn, when there is counting, there is logic. Therefore, patterns in nature often align with logical set-ups. Familiar patterns such as 2, 4, 6, and 8 are among the first patterns learned in the early years. Patterns can take many forms including sequential, spatial, temporal, and linguistic. For example, the most basic pattern is the sequence of dates in the calendar, from 1 to 30. In the world, regularity is the fact that the same thing always happens in the same circumstances. The patterns in nature are regularities of form found in the natural world and can be mathematically modeled. Examples of natural patterns include symmetries, tree spirals, meanders, weaves, foams, tessellations, cracks, and stripes. A geometric pattern is a type of pattern created from geometric shapes, often repeated like a wallpaper design.

EXAMPLES OF PATTERNS IN NATURE

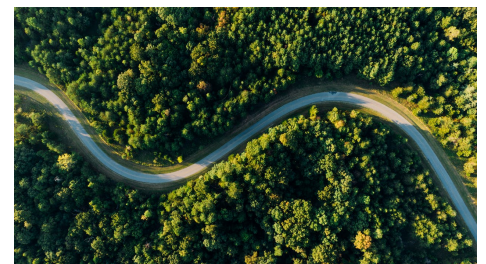
- Symmetry - refers to the property of an object remaining unchanged after undergoing certain transformations, such as reflection, rotation, or scaling. This means that the dimension, proportions, and arrangement of the object are in agreement and remain consistent. In other words, symmetry is the property of an object that remains unchanged under specific operations or transformations. It is a fundamental concept in many branches of mathematics, including geometry, algebra, and topology and it plays an important role in physics, chemistry, and other natural sciences.



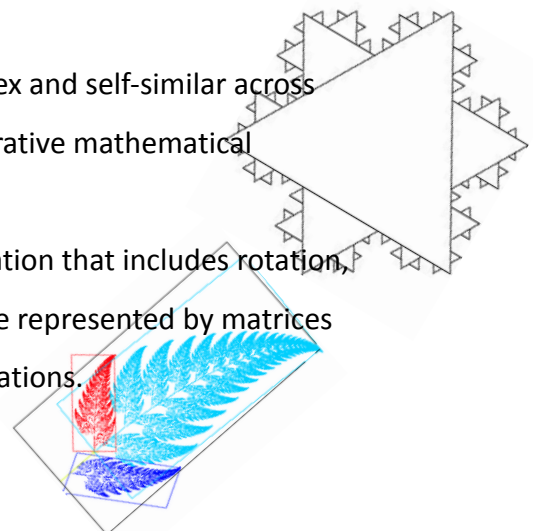
- Spiral - is a curve that emanates from a central point, and as it moves away from that point, it revolves around it. The curve can be described by a set of parametric equations, polar equations, or implicit equations. There are many different types of spirals, including the Archimedean spiral, the logarithmic spiral, and the Fermat spiral. Spirals can be found in many natural phenomena such as the shapes of seashells, the shape of galaxies, and the pattern of a pine cone. It is also an important shape in art and design and is often used in architecture and engineering.



- Meander - in geography, a meander is one of a series of regular, sinuous curves, bends, loops, turns, or windings in the channel of a river, stream, or another water source.

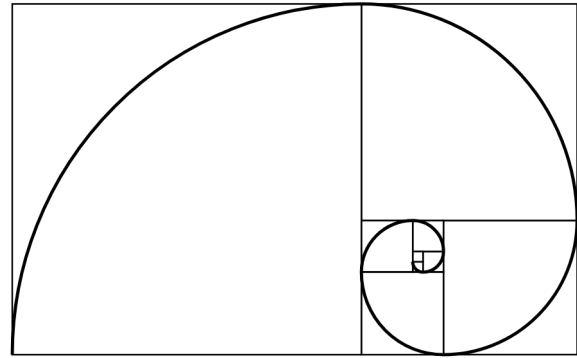


- Wave - in physics, a wave is a disturbance that transfers energy through matter or space, with little or no mass transport.
- Tessellations - a flat surface refers to the tiling of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps.
- Fractal - is a never-ending pattern that is infinitely complex and self-similar across different scales. Fractals are typically generated using iterative mathematical algorithms, but they can also be found in nature.
- Affine Transformation - is a class of geometric transformation that includes rotation, reflection, scaling, translations, and shearing. They can be represented by matrices and can be composed to create more complex transformations.



The Fibonacci Sequence

Another pattern found in the world is the Fibonacci numbers. These numbers, known as nature's numbering system, have a significant presence in the natural world and are named after the Italian mathematician Leonardo Fibonacci. In mathematics, the



Fibonacci numbers are a sequence of integers characterized by the fact that every number after the first two is the sum of the two preceding ones: 1,1,2,3,5,8,13,21,34,55,89,144, and so on. The Fibonacci sequence appears in many natural phenomena, such as the branching patterns of trees, the arrangement of leaves on a stem, the pattern of seeds in a sunflower, and the spiral patterns in pinecones and seashells.

Examples of Fibonacci Sequence

Seed head	Pine cones	Tree branches	Shells	Galaxies
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Importance of Mathematics in Life

Mathematics plays a vital role in our daily lives and is present in many aspects of our daily activities. Some examples of its importance include

- 1. Restaurant Tipping:** Mathematics is used to calculate the appropriate amount to tip based on the cost of the meal and the level of service.
- 2. Netflix film viewing:** Recommendation algorithms used by Netflix use mathematical concepts such as probability and statistics to suggest films to viewers.
- 3. Calculating bills:** Mathematics is used to calculate the cost of goods and services, including taxes and discounts.
- 4. Computing test scores:** Mathematics is used to calculate test scores, grades, and other academic progress measurements.
- 5. Tracking career:** Math is used in many fields such as finance, accounting, engineering, and science, to track and analyze data to make informed decisions for career development.

Nature of Mathematics

According to the American Association for the Advancement of Science (1990), mathematics relies on both logic and creativity, and it is pursued for both practical purposes and its intrinsic interest. This means that mathematics is a field that requires both logical thinking and problem-solving skills, as well as the ability to think creatively and abstractly. The application of mathematics can be found in many fields, including science, engineering, finance, and technology, as well as in everyday life. Additionally, the study of mathematics can also be pursued for its own sake, as an intellectual pursuit that can be appreciated for its beauty and elegance. Mathematics is a subject that has many practical applications and it can be used to solve real-world problems. It's also an important tool for critical thinking and problem-solving which are valuable skills in any profession.

1. *Patterns and Relationships* - Mathematics is the study of patterns and relationships, it is a theoretical discipline that explores the possible connections and interactions between abstract concepts, independent of their real-world counterparts. It is the science of patterns and how different elements are related to one another, and how they can be described and analyzed using mathematical concepts and structures.
2. *Mathematics, Science, and Technology* - are closely related fields. The abstract nature of mathematics makes it an ideal tool for understanding and working with the natural laws that govern the physical world, and it is a fundamental component of science and technology. Because of its abstractness, mathematics can be applied in a wide range of fields, and it is considered a universal language of sorts, that is not limited to a specific field of human thought.
3. *Mathematical Inquiry* - the process of solving problems that people encounter in their daily lives to achieve a sense of peace and understanding.
4. *Abstraction and Symbolic Representation* - the process of abstraction often involves recognizing similarities between two or more objects or events, and creating a symbolic representation to describe them. This process of abstraction allows mathematicians to create general concepts and principles that can be applied to a wide range of problems.
5. *Manipulating Mathematical Statements* - involves combining and recombining symbols, which have been selected to represent abstract concepts, according to

precisely defined rules. This process allows for the manipulation of mathematical statements and the solving of mathematical problems.

6. Application - models can be used to gain insights about a subject or phenomenon by simulating and analyzing its behavior through mathematical processes.

ROLES OF MATHEMATICS IN SOME DISCIPLINES



Mathematics is offered in many college courses and is found in nearly every curriculum because its theories and applications are needed in many fields of study and many workplaces. According to Angel Rathnabai (2014), Mathematics is not only about working with numbers or computation, but it also involves forming generalizations, seeing relationships, and developing logical thinking and reasoning. This broad range of applications of mathematics makes it a valuable tool for understanding and solving problems in a wide range of fields, including science, engineering, finance, economics, and many more. As a result, it's important to have a strong foundation in mathematics to be successful in any profession.

1. *Mathematics in Physical Sciences* - In the physical sciences, mathematics plays a crucial role in understanding the principles and laws of physics. Mathematical equations and calculations are used to express and explain the rules of physics, giving them a precise and workable form. Mathematics is present at every step of the process in physics, from the formulation of theories to the interpretation of experimental data.
2. *Mathematics in Chemistry* - The importance of mathematics in chemistry, specifically in physical chemistry, cannot be overstated, particularly in advanced topics such as quantum mechanics and statistical mechanics.
3. *Mathematics in Biological Science* - The application of mathematics in the field of biological science, also known as biomathematics, is a vast and exciting field with many open problems to explore. These problems include mathematical genetics, mathematical ecology, mathematical neuropsychiatry, and the development of computer software for specific biological and medical issues.

4. *Mathematics in Engineering and Technology* - The application of mathematics in the field of engineering and technology is well-established and is considered to be a fundamental aspect of engineering. It is used to solve complex problems, design and analyze systems, and make predictions about the behavior of these systems. From the design of bridges and buildings to the development of new technologies, mathematics plays a crucial role in the field of engineering and technology.
5. *Mathematics and Agriculture* - The application of mathematics in agriculture is significant and extensive. The use of mathematical concepts and techniques is crucial in many aspects of agriculture, including measurements of land and area, calculation of investment and expenses, determination of returns and income, calculation of production per unit area, cost of labor and time, and calculation of seed rate. These mathematical calculations and analyses are essential for making informed decisions and improving efficiency in agriculture.
6. *Mathematics and Economics* - The use of mathematical concepts and techniques are becoming increasingly important in personal and social activities. A strong understanding of these fundamental processes is essential for clear and effective decision-making.
7. *Mathematics and Psychology* - The application of mathematics to the field of psychology, as stated by educationist Herbart, is not only possible but also necessary. The use of mathematical concepts and techniques in experimental psychology has become increasingly important as it allows for a deeper understanding of factors such as intelligence quotient, standard deviation, mean, median, mode, correlation coefficients, and probable error. This approach has helped to make psychology more precise and accurate in its findings.
8. *Mathematical and Actual Science, Insurance, and Finance* - Actuaries use mathematical and statistical methods to assess and manage risk in the financial industry. They analyze data and make predictions about future events, particularly regarding pensions and long-term financial solvency for Engineering
9. *Mathematics and Archaeology* - archeologists employ mathematical and statistical methods to analyze data from archaeological surveys to uncover patterns and gain insights into past human behavior.

- 10. *Mathematics and Logic*** - D'Alembert states that geometry is a practical application of logic, as it applies to reason simply and sensibly. Pascal adds that logic borrows the rules of geometry and that while logicians aim to avoid error, only geometers truly achieve it, and that without the science of geometry, there can be no true understanding.
- 11. *Mathematics in Arts*** - The application of mathematics in the field of art is a fascinating area of study. Both mathematics and art can be used to express similar ideas and concepts, and it is often said that the universe is written in the language of mathematics, with its characters being geometric figures such as triangles and circles. This connection between mathematics and art can be seen in the use of geometric shapes and patterns in art, as well as in the use of mathematical principles in the creation of art.
- 12. *Mathematics in Music*** - The application of mathematics in music, as stated by Leibnitz, a renowned mathematician, is a subtle yet powerful tool that can be used to understand the underlying structure and patterns in music. Music theorists often use mathematical concepts to analyze and comprehend musical compositions, and to communicate new perspectives and ways of listening to music.
- 13. *Mathematics in Philosophy*** - The application of mathematics in the field of philosophy is significant. Mathematics plays a crucial role in shaping philosophical thought, as stated by Herbart, a renowned educationist who believed that mathematics serves as a protective barrier against the potential dangers of philosophy. It helps to sharpen the mind and develop critical thinking, allowing for a deeper understanding of philosophical concepts and ideas. In addition, mathematics is often used in the formalization of philosophical theories and arguments.
- 14. *Mathematics in the Social Networks*** - The application of mathematics in the field of social networks involves the use of various mathematical techniques such as graph theory, text analysis, multidimensional scaling, and cluster analysis to analyze data from different social networks. These techniques are used to understand patterns, connections, and trends in the data, make predictions and identify opportunities for further research. This field is rich with open, challenging, and fascinating problems that are yet to be explored.

- 15. *Mathematics in Political Science*** - The field of mathematical political science uses mathematical methods to analyze past election results, study changes in voting patterns, and investigate the impact of various factors on voting behavior, including the switching of votes among political parties. Additionally, mathematical models are used to study conflict resolution in politics.
- 16. *Mathematics in Linguistics*** - Mathematics in linguistics involves the application of mathematical concepts to the study of language. This includes the use of structure and transformation, which are fundamental concepts in both mathematics and linguistics, to analyze and understand the properties of language. The use of mathematical methods in linguistics can help to uncover the underlying patterns and regularities in language, and to develop models and theories of language structure and use.
- 17. *Mathematics in Management*** - The application of Mathematics in management presents a stimulating challenge for those with creative minds. It requires individuals to think outside the box and not simply rely on routine methods.
- 18. *Mathematics in Computers*** - is a crucial area that involves the application of mathematical theories to the development and advancement of computer technology. This includes the use of formal mathematical methods and concepts to design and analyze algorithms, data structures, and other fundamental components of computer systems. The field is constantly evolving and presents ongoing challenges and opportunities for research and innovation.
- 19. *Mathematics in Geography*** - plays a crucial role in the field of geography, where it is used to provide scientific and mathematical descriptions of the Earth and its position in the universe. From understanding the dimension and magnitude of the Earth to its location in the universe, the formation of days and nights, lunar and solar eclipses, latitude and longitude, maximum and minimum rainfall, and many other phenomena, mathematics is a fundamental tool in the study of geography. These concepts and principles are used to analyze, understand and describe the Earth's physical and human features, and make predictions about future trends and patterns.

Appreciating Mathematics as Human Endeavor

Mathematics is a fundamental aspect of many professions and fields of study. To fully appreciate its importance, it is essential to understand it as a human endeavor. From accountants working on taxes and planning for future years, to agriculturists determining the proper amounts of fertilizer and water to produce bountiful crops, to architects designing buildings for structural integrity and beauty, mathematics plays a crucial role in these fields. Similarly, biologists use proportions and statistics to study nature, chemists use mathematical concepts to develop new products and processes, and computer programmers use mathematical algorithms to create software programs. Engineers build structures and systems, lawyers argue cases using logical reasoning, and managers analyze productivity. In the medical field, doctors use mathematical concepts to understand the dynamic systems of the human body, and nurses carry out detailed instructions given by doctors. Meteorologists forecast the weather, military personnel carry out a variety of tasks, and politicians make complex decisions within the confines of the law, public opinion, and budget constraints. Salespeople operate under a profit model, technicians repair and maintain technical gadgets, and tradesmen estimate job costs and use technical math skills specific to their fields. Overall, mathematics is an essential tool in many areas of life and it is important to appreciate its value as a human endeavor.

How can Mathematics be so universal?

Just like science, math is also everywhere. It explains everything to us just like your daily routine, the time and the date of execution, and also whatever you buy in the market. It has become essential for us to use numbers or digits in our daily lives because, without them, we are just a product of nothing, we will become stupid because we don't know how to count or use math. Everywhere we see all kinds of examples of people who use math in different ways not only in the country they live in but also expand out in the entire universe. Scientists use math to tell what time it is or to measure how far exactly a star is. That is why it is called universal because it can be used in a lot of ways. - Kevin Mamaspas

Lesson 2

Mathematical Language and Symbols

Mathematical language and symbols are a way to express mathematical concepts and ideas clearly and concisely. This language is composed of a set of symbols, such as numbers, letters, and operators, which are used to represent mathematical operations and relationships. For example, the symbol "+" represents an addition, while the symbol "=" represents equality. The symbols and notation used in mathematics have been developed over time to make it easier to understand and communicate mathematical ideas. They allow mathematicians to express complex concepts in a compact and easily understandable form, and they also allow for the efficient manipulation of mathematical expressions. The use of mathematical language and symbols also enables mathematical proofs and solutions to be written in a precise and unambiguous way, which is important for the development of mathematical knowledge.

The language of Mathematics makes it easy to express the kinds of symbols, syntax, and rules that mathematicians like to do characterized by the following:

- a. PRECISE** - the four basic operations in mathematics are addition, subtraction, multiplication, and division. These operations form the foundation of arithmetic and are used to perform calculations and solve mathematical problems. Addition is the process of combining two or more numbers to find their sum. For example, $2 + 3 = 5$. **Subtraction** is the process of finding the difference between two numbers. For example, $5 - 2 = 3$. **Multiplication** is the process of repeated addition. For example, $2 \times 3 = 6$. The **division** is the process of finding how many times one number is contained within another number. For example, $6 \div 2 = 3$. These basic operations are essential for solving mathematical problems and are used in a wide variety of mathematical concepts, including algebra, geometry, and calculus. They are also used in everyday life, such as in budgeting, cooking, and many other activities.
- b. CONCISE** - In summary, being concise in mathematics is about expressing ideas clearly and effectively, using the most appropriate notation and symbols, writing proofs and solutions in a logical and easy-to-follow manner, and solving problems efficiently.
- c. POWERFUL** - Being powerful in mathematics refers to the ability to solve complex problems and analyze mathematical concepts using a wide range of mathematical tools and methods. It can also mean the ability to apply mathematical knowledge to solve problems in a variety of fields and disciplines.

Mathematical expression - A mathematical expression is a combination of numbers, variables, and operators that represent a mathematical value or relationship. For example;

$2 + 3 = 5$. In this example, "2" and "3" are numbers, "+" is the operator that represents an addition, and "5" is the result of the mathematical expression. Another example of a mathematical expression is:

$y = 2x + 3$. In this example, "y" and "x" are variables, "2" and "3" are numbers, "=" is the operator that represents equality, and "y" is the result of the mathematical expression, which represents the equation of a straight line.

A sample of mathematical expressions can be:

1. The area of a circle = πr^2
2. The quadratic equation: $ax^2 + bx + c = 0$
3. The Pythagorean theorem: $a^2 + b^2 = c^2$
4. The distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

All these examples show different mathematical expressions that represent different mathematical concepts, relationships, or formulas. These mathematical expressions can be used to solve problems and understand the mathematical concepts they represent.

Math Symbols

$+$ plus/positive	$-$ minus/negative	\times times/multiply	\div / divide
$=$ equality	\neq inequality	\approx approximately equal	\pm plus or minus
$<$ is less than	\leq is less than or equal to	$>$ is greater than	\geq is greater than or equal to
∞ infinity	$!$ factorial	\emptyset empty set	$\%$ percent
π pi	\therefore therefore	\because because	\sum sum of
\int integral	$ x $ absolute value of x	\sim is similar to	\parallel is parallel to
$\sqrt{\quad}$ square root	α alpha	β beta	\equiv is congruent to

Mathematical Convention - A mathematical convention is a standard or agreed-upon way of representing mathematical concepts or ideas. These conventions help to ensure consistency and clarity in mathematical communication, and they are used to make mathematical notation and symbols more easily understood.

Some examples of mathematical conventions include:

1. **Order of operations:** This convention is used to specify the order in which mathematical operations should be performed. The order of operations is typically represented by the acronym PEMDAS, which stands for Parentheses, Exponents, Multiplication, Division, and Addition Subtraction.

2. *Notation for sets*: This convention is used to specify the notation for sets of numbers or objects. For example, a set of numbers is typically represented by curly braces {}.
3. *Notation for functions*: This convention is used to specify the notation for functions. For example, a function is typically represented by the letter "f" and the independent variable is represented by the letter "x".
4. *Notation for derivatives and integrals*: This convention is used to specify the notation for derivatives and integrals. For example, the derivative of a function is typically represented by the symbol " $\frac{d}{dx}$ " or by the prime symbol " $f'(x)$ ".
5. *Notation for series and sequence*: This convention is used to specify the notation for series and sequence. For example, a series is typically represented by the summation sign, and a sequence is represented by a subscripted variable such as " a_n ".

These are just a few examples of mathematical conventions, and there are many more conventions that are used in different areas of mathematics. Adhering to mathematical conventions helps to ensure consistency and clarity in mathematical communication and makes it easier for others to understand and follow mathematical reasoning.

B brackets	O order	D divide	M multiply	A add	S subtraction
()	$\sqrt{x \times 2}$	\div	\times	+	-
P parenthesis	E exponents	M multiply	D divide	A add	S subtract

The order of operations, also known as BODMAS or PEMDAS, is a set of rules that prioritize the sequence of operations in a mathematical expression. It is important to follow these rules to ensure that mathematical expressions are evaluated correctly. The acronym BODMAS stands for Bracket, Of, Division, Multiplication, Addition, and Subtraction. PEMDAS stands for Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction. Both acronyms refer to the same order of operations and the order of priority of the operation. As you described, the order of operations starts with simplifying everything inside the parentheses first, then simplifying an exponential number, then performing any multiplication and division from left to right, and finally performing any addition and subtraction from left to right.

It's worth noting that when the operations have the same priority, such as multiplication and division or addition and subtraction, the operations should be performed from left to right. By following these rules, one can avoid confusion and errors in solving mathematical problems and also it helps to ensure that mathematical expressions are evaluated consistently and correctly.

The four basic concepts of Mathematics

Sets	Relation	Function	Binary Function
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Sets - In mathematics, a set is a collection of distinct objects, known as elements or members of the set. Sets are used to group and organize mathematical objects, and they are represented using set notation. The notation typically used to represent a set is a pair of curly braces $\{\}$. The elements of the set are listed inside the curly braces, and they are separated by commas. For example, the set of natural numbers can be represented as $\{1, 2, 3, 4, 5, 6, \dots\}$. The set of even numbers can be represented as $\{2, 4, 6, 8, 10, \dots\}$.

There are different types of sets in mathematics, such as:

- Finite sets: sets that have a limited number of elements
- Infinite sets: sets that have an infinite number of elements
- Empty set: a set that has no elements
- Subsets: a set that contains elements of another set
- Power set: the set of all subsets of a set

Sets are used in a wide range of mathematical concepts, including number theory, set theory, algebra, and geometry. They are also used in everyday life, such as in statistics and computer science. Sets also have operations such as union, intersection, complement, and Cartesian product. These operations are used to combine or compare sets and to extract new sets from the given sets. In summary, a set is a collection of distinct objects or elements, represented using set notation, and it is used to group and organize mathematical objects. It is a fundamental concept in mathematics and it has many uses in different areas of mathematics and other fields.

In mathematics, a relation is a set of ordered pairs of elements from two sets, called the domain and the range, that defines a specific connection or relationship between the elements. A relation is often represented by a mathematical function, which assigns an output to each input value. There are different types of relations in mathematics, such as:

- One-to-one relations: a relation where each element in the domain is related to exactly one element in the range.
- Many-to-one relations: a relation where multiple elements in the domain are related to the same element in the range.
- One-to-many relations: a relationship where one element in the domain is related to multiple elements in the range.
- Many-to-many relations: a relation where multiple elements in the domain are related to multiple elements in the range.

In summary, a relation in mathematics is a set of ordered pairs that define a connection or relationship between elements from two sets (domain and range). It can be represented by a mathematical function, graph, or table and it can be used to understand and analyze different mathematical concepts and real-world situations.

Lesson 3

Problem-Solving and Reasoning

Problem-solving and reasoning are essential skills in mathematics, as well as in many other fields. They involve the ability to analyze a problem, identify key information and concepts, and apply mathematical knowledge to find a solution. Problem-solving in mathematics usually starts with understanding the problem, which means identifying the mathematical concepts and operations involved, the unknowns, and the given information. Once the problem is understood, it's essential to plan a strategy to solve it, which means identifying the steps and methods that will be used to find the solution. After that, it's time to execute the plan and use mathematical concepts and methods to solve the problem. It's important to be careful and accurate when solving the problem and to check the solution to make sure it is correct. Reasoning in mathematics is the process of applying logical thinking to arrive at a conclusion or proof. It's a critical skill in mathematics and it is used to justify mathematical statements, prove mathematical theorems, and draw logical conclusions from mathematical information. Proper mathematical reasoning involves using logical deduction and induction, making connections between different mathematical concepts, and using mathematical language and notation to express ideas clearly and precisely. In summary, problem-solving and reasoning are essential skills in mathematics that involve the ability to analyze a problem, plan a strategy, execute the plan, and use logical thinking to arrive at a conclusion or proof. These skills are not only important in mathematics but also in many other fields and daily life.

Kinds of Reasoning

There are several types of reasoning in mathematics, including:

1. *Deductive reasoning*: This type of reasoning involves starting with a general statement or hypothesis and using logical reasoning to arrive at a specific conclusion. For example, if we know that all rectangles have four right angles, we can deduce that a specific rectangle has four right angles.
2. *Inductive reasoning*: This type of reasoning involves starting with specific examples or observations and using them to make general statements or conclusions. For example, after observing several triangles and noticing that the sum of the angles is always 180 degrees, we can inductively conclude that all triangles have a sum of angles equal to 180 degrees.
3. *Abductive reasoning*: This type of reasoning involves starting with an observation or outcome and then using it to infer the best explanation or hypothesis. For example, if we observe that a certain disease outbreak is affecting only people living in a certain area, we can abductively infer that the outbreak is caused by something specific to that area.

4. *Analogical reasoning*: This type of reasoning involves using a comparison or analogy to understand a new concept or solve a problem. For example, to understand the concept of congruent figures, one could use the analogy of two people wearing the same clothes.
5. *Counterfactual reasoning*: This type of reasoning involves imagining what would happen if something were different from what it is and then drawing a conclusion from that. For example, if we know that a certain function is increasing, we can use counterfactual reasoning to infer what would happen if the function were decreasing.
6. *Retroductive reasoning*: This type of reasoning is a combination of abductive and deductive reasoning. It starts with a set of observations and then uses it to infer a hypothesis, which is then tested and refined through further observation and experimentation.

These are just a few examples of the types of reasoning used in mathematics, and many other types are used in different areas of mathematics and other fields. The ability to use different types of reasoning is essential for solving mathematical problems and understanding mathematical concepts.

What next possible?



Daddy Jones uses his logical and reasoning to make a decision. Rawr!

If-then Statements and Converses

An "if-then" statement, also known as a conditional statement, is a type of mathematical statement that expresses a relationship between two events or statements. It is written in the form "if A, then B," where A is called the antecedent and B is called the consequent. The statement is read as "if A is true, then B is also true." For example, the statement "If x is a prime number, then x is odd" is an if-then statement. The antecedent is "x is a prime number" and the consequent is "x is odd." The converse of an if-then statement is formed by switching the antecedent and consequent. The converse of the statement above would be "If x is odd, then x is a prime number."

It's important to note that the converse of an if-then statement is not always true. For example, the statement "If x is odd, then x is a prime number" is not true, because there are odd numbers that are not prime (e.g. 9). A statement that is true in both its original form and its converse form is called a biconditional statement. An example of a biconditional statement is " x is a prime number if and only if x is odd". In summary, an if-then statement expresses a relationship between two events or statements, the converse of an if-then statement is formed by switching the antecedent and consequent, not all if-then statements have a true converse, and a statement that is true in both its original form and its converse form is called a biconditional statement.

Polya's 4 steps in Problem Solving and Problem Solving Strategies

Polya's four steps in problem-solving are as follows:

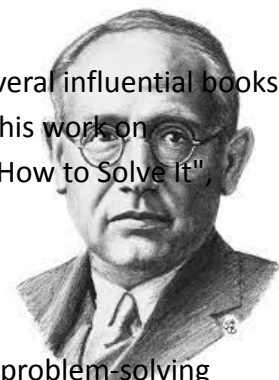
1. Understand the problem: Read the problem carefully and try to understand what is being asked. Identify the unknown and the data given in the problem.
2. Devise a plan: Come up with a plan or strategy to solve the problem.
3. Carry out the plan: Put the plan into action and solve the problem.
4. Evaluate the solution: Check the solution to make sure it is correct and makes sense.

Problem-solving strategies include:

1. Breaking the problem down into smaller parts
2. Using a trial-and-error approach
3. Making a list or chart
4. Using an equation or formula
5. Working backward
6. Using a visual representation
7. Using a guess-and-check method
8. Using a logical reasoning approach
9. Consulting a reference or resource
10. Asking for help or collaborating with others.

Who is Polya?

George Polya was a Hungarian mathematician and author of several influential books on problem-solving and mathematical education. He is best known for his work on heuristics, which are strategies for solving problems, and for his book "How to Solve It", which is widely used in math and computer science education.



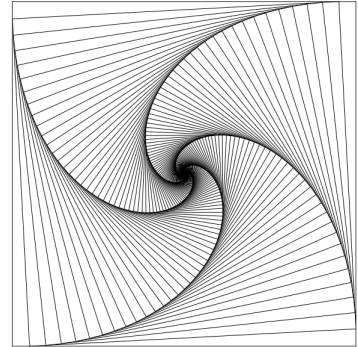
Always remember;

In summary, critical thinking, the scientific method, and Polya's problem-solving method are effective ways to arrive at accurate conclusions by evaluating information and arguments, formulating hypotheses and testing them, breaking down a problem into smaller subproblems, and identifying patterns and connections. Using these methods in daily life helps make informed decisions and avoid being misled by inaccurate information.

Lesson 4

Geometric Design

Geometric design in mathematics is the study of the use of geometric shapes, figures, and patterns in various fields, such as architecture, engineering, and computer graphics. It involves the application of mathematical principles, such as measurement, symmetry, and optimization, to the design and creation of structures and shapes. In the modern world, geometric design is used in a wide range of industries, including transportation, construction, and product design. It also plays an important role in computer-aided design (CAD) and computer-aided manufacturing (CAM) systems, as well as in the creation of virtual and augmented reality environments.



Geometric concepts are fundamental principles and ideas used in the study of geometry. Some common geometric concepts include:

- **Points:** A point is an element of a geometric space that represents a location and has no size or shape.
- **Lines:** A line is a one-dimensional geometric object that extends infinitely in both directions.
- **Angles:** An angle is the measure of the amount of rotation between two lines that share a common point, or vertex.
- **Triangles:** A triangle is a polygon with three sides and three angles.
- **Quadrilaterals:** A quadrilateral is a polygon with four sides and four angles.
- **Circles:** A circle is a two-dimensional geometric object consisting of all points that are the same distance from a central point.
- **Perimeter:** The perimeter of a shape is the sum of the lengths of all its sides.
- **Area:** The area of a shape is the amount of space it occupies.
- **Volume:** The volume of a three-dimensional shape is the amount of space it takes up.
- **Transformations:** Transformations are changes made to the position, size, or shape of a geometric figure.
- **Symmetry:** Symmetry is the property of a shape that remains unchanged under certain operations or transformations.
- **Congruence:** Congruence refers to the property that two shapes are identical in size and shape.
- **Similarity:** Similarity refers to the property that two shapes are the same shape but not necessarily the same size.

These concepts are used in various branches of mathematics, physics, engineering, and many more fields.

Analyzing geometric shapes according to the level of geometric thinking can be done through the following steps:

1. **Recognition:** The first step is to identify and recognize the basic properties of the shape, such as its name, number of sides, angles, and other characteristics. This level of thinking is known as "recognition" and is considered the most basic level of geometric thinking.
2. **Analysis:** Once the basic properties of the shape have been identified, the next step is to analyze the shape in more detail. This might involve measuring and comparing its sides, angles, and other characteristics to other shapes. This level of thinking is known as "analysis" and is considered a more advanced level of geometric thinking.
3. **Synthesis:** The next level of geometric thinking is "synthesis" which is creating new shapes or figures using the known ones, this could be done by combining, dividing, or modifying the shapes in different ways.
4. **Evaluation:** The final level of geometric thinking is "evaluation" which is to make judgments about the geometric figures based on the knowledge acquired from the previous steps. This can include making comparisons, drawing conclusions, or identifying patterns and relationships among the shapes.

It's important to note that individuals may think differently and may not follow a linear progression through these levels and that some levels of thinking may be more developed than others.

The properties of the levels of geometric thinking are as follows:

1. **Recognition:** This level of thinking involves the ability to identify and recognize basic properties of shapes, such as their name, number of sides, angles, and other characteristics. It is the most basic level of geometric thinking and is often associated with memorization and rote learning.
2. **Analysis:** Analysis is the ability to examine shapes in more detail, using techniques such as measurement, comparison, and observation. This level of thinking involves understanding the relationships between different parts of a shape, such as the angles in a triangle. It is considered a more advanced level of geometric thinking, as it requires a deeper understanding of the properties of shapes.
3. **Synthesis:** This level of thinking involves the ability to create new shapes or figures using known ones, this could be done by combining, dividing, or modifying the shapes in different ways. It requires a deep understanding of the properties of shapes and the ability to apply this knowledge in new and creative ways.
4. **Evaluation:** This level of thinking involves making judgments about geometric figures based on the knowledge acquired from the previous steps. It requires the ability to identify patterns and relationships among shapes, make comparisons, and draw conclusions. It is the highest level of geometric thinking, as it requires a deep understanding of the properties of shapes and the ability to apply this knowledge critically and reflectively.

It's worth noting that some individuals may think differently and may not follow a linear progression through these levels and that some levels of thinking may be more developed than others.

There are several **types of geometric transformations**, including:

1. Translation: A translation is a transformation in which a shape is moved to a different position on the plane without changing its size or orientation.
2. Rotation: A rotation is a transformation in which a shape is turned around a fixed point, known as the center of rotation. The angle of rotation and the direction of rotation determines the final position of the shape.
3. Reflection: A reflection is a transformation in which a shape is reflected across a line of symmetry, known as the line of reflection. The shape is flipped over the line of reflection, creating a mirror image of the original shape.
4. Dilation: A dilation is a transformation in which a shape is scaled (enlarged or reduced) by a certain factor. The center of dilation is a fixed point, and the scale factor determines the amount of change in size.
5. Shearing: A shearing transformation is a transformation in which a shape is shifted in a direction parallel to one of its axes.
6. Glide Reflection: A glide reflection is a combination of a translation and a reflection across a line of symmetry. The shape is reflected and then translated by a certain distance.
7. Composition of function: A composition of function is the application of two or more transformations on a shape in a specific order.

These transformations are used in various branches of mathematics, physics, engineering, and many more fields. They are also commonly used in computer graphics and animation to create and manipulate images and 3D models.

How to use Geometric Patterns Design?

Geometric patterns and designs can be used in a variety of ways, some of which include:

1. Architecture: Geometric patterns and designs are often used in architecture to create aesthetically pleasing and functional structures. They can be used in the design of buildings, bridges, and other structures to create symmetry, balance, and a sense of movement.
2. Interior design: Geometric patterns and designs can be used in interior design to create a sense of visual interest and movement in space. They can be used on floors, walls, and ceilings to create patterns and designs that complement the overall aesthetic of the space.
3. Product design: Geometric patterns and designs are often used in product design to create visually appealing and functional objects. They can be used in the design of furniture, textiles, and other products to create patterns and designs that are both visually interesting and functional.

4. Advertising: Geometric patterns and designs are often used in advertising to create visually striking and memorable images. They can be used in the design of logos, posters, and other advertising materials to create a sense of visual interest and movement.
5. Fine arts: Geometric patterns and designs are widely used in fine arts, such as painting and sculpture, to create visual interest and movement. They can be used to create abstract or non-representational art, and also to create patterns and designs in traditional art forms.
6. Computer graphics and animation: Geometric patterns and designs are widely used in computer graphics and animation to create and manipulate images and 3D models. They can be used to create complex shapes, patterns, and designs that can be rendered in real-time for interactive applications such as video games, and also used in film and television animation.

It's important to note that the use of geometric patterns and designs is not limited to these examples, and they can be used in many other fields as well.

Lesson 5

The Codes

Codes in mathematics refers to the use of mathematical algorithms and techniques to encrypt and decrypt information. These codes are used to secure sensitive information, such as financial data or personal information so that it can be transmitted or stored securely. In the modern world, codes and encryption play a crucial role in keeping information secure in various fields such as e-commerce, banking, and government communications.

There are many different types of codes and encryption methods used in modern mathematics, including:

1. Symmetric-key encryption: This type of encryption uses the same key for both encrypting and decrypting the information.
2. Public-key encryption: This type of encryption uses two different keys, one for encryption and one for decryption. The encryption key is made publicly available, while the decryption key is kept private.
3. Hash functions: These are mathematical functions that take an input (or 'message') and return a fixed-size string of characters, which is the 'digest' or 'hash value'. They are used to ensure the integrity and authenticity of data by detecting changes to the original message.
4. Error-correcting codes: These codes are used to detect and correct errors that may occur during the transmission or storage of data.
5. Steganography: This is the practice of hiding a message within another message or image.

These codes are used in various fields such as computer science, network security, cryptography, and telecommunications. With the increasing reliance on digital communications and online transactions, the importance of codes and encryption in mathematics is likely to continue to grow in the future.

What are Binary codes?

- Binary codes are a way of representing information using only two symbols, typically 0 and 1. These codes are used in digital systems, such as computers and digital devices, to represent and manipulate data. Binary code is also known as "machine code" or "computer code" as it is the lowest level of programming language and it is the language that the computer understands.

```
01010100 01101000 01101001 01110011
00100000 01101001 01110011 00100000
01110100 01101000 01100101 00100000
01110100 01110101 01110100 01101111
01110010 01101001 01100001 01101100
00100000 01110100 01101111 00100000
01101100 01100101 01100001 01110010
01101110 00100000 01100010 01101001
01101110 01100001 01110010 01111001
00101110 00100000 01001001 00100000
01101000 01101111 01110000 01100101
00100000 01111001 01101111 01110101
00100000 01100101 01101110 01101010
01101111 01111001 00100000 01101001
01110100 00100001
```

In a binary code, each digit (bit) represents a power of 2, starting from 2^0 for the rightmost bit. For example, the binary number "101" represents the decimal number 5 ($12^2 + 02^1 + 1*2^0$). This allows computers to represent and process data in a way that is simple and efficient, as all data can be represented as a series of ones and zeroes. Binary codes are used in many areas of computer science and engineering, including computer programming, data storage, and digital communications. They are also used in digital logic and digital electronics, such as digital circuits, digital signal processing, and digital control systems. In short, binary codes are a way of representing data in a digital system, it's a language that the computer understands and it's the fundamental building block of the digital world.

Binary codes are a type of code that uses only two symbols, typically 0 and 1, to represent information. These codes are used in digital systems, such as computers and digital devices, to represent and manipulate data.

Some examples of binary codes include:

1. ASCII (American Standard Code for Information Interchange): This code is used to represent characters and symbols in a computer. Each character is represented by a unique combination of 8 binary digits (bits). For example, the letter "A" is represented by the binary code 01000001.
2. Unicode: This code is similar to ASCII but it uses 16 bits to represent characters, it includes a much wider range of characters, including those from non-English languages.
3. Gray Code: This code is used to minimize the number of changes required when transitioning from one number to another. It is commonly used in digital to analog converters and encoders.

4. Huffman coding: this method is used for lossless data compression, it assigns short codewords to more frequently occurring characters, and longer codewords to less frequently occurring characters.
5. Error-correcting code: these codes are used to detect and correct errors that may occur during the transmission or storage of data, such as Hamming code and Reed-Solomon codes.

These are just a few examples of binary codes that are used in modern technology, there are many other codes and coding schemes used in different fields such as image and video compression, data storage, and communications.

An example of converting a letter to binary code is using the ASCII (American Standard Code for Information Interchange) code.

The ASCII code is a standardized system that assigns a unique binary code to each character and symbol on a keyboard. For example, the letter "A" is represented by the binary code 01000001. This can be broken down as follows:

- The first bit is 0, indicating that it is not an extended character
- The next 7 bits (from the 2nd bit to the 8th bit) are 1000001, which corresponds to the decimal number 65, which represents the letter "A" in the ASCII table.

Another example, let's say you want to convert the letter "Z" to binary.

The ASCII code for the letter "Z" is 90 in decimal, which is 1011010 in binary.

It's worth noting that there are different ways of encoding letters to binary, such as UTF-8 or UTF-16, which uses more than 8 bits to represent a character and includes a much wider range of characters, including those from non-English languages.

What is the difference between BITS and BYTES?

- Bits and bytes are units of measurement that are used to describe the amount of data or memory in a computer system.

A bit (binary digit) is the basic unit of information in a computer. It can have two possible values, 0 or 1. It is the smallest unit of data that a computer can process. It is used to represent the binary data and the on-and-off states of the computer system. A byte, on the other hand, is a unit of measurement that is made up of 8 bits. One byte can represent a number between 0 and 255 in decimal or between 00 and FF in hexadecimal. A byte can represent a character in a text file, an image in a picture file, or a sound in a music file. In summary, a bit is the basic unit of digital data, it's a single digit in the binary numeral system. A byte is a unit of digital information that typically consists of eight bits, it's a unit of measurement for the amount of data that can be stored or processed.

The several ways to the computing of binary

There are several ways to perform arithmetic operations on binary numbers, including:

Addition: Addition in binary works the same as an addition in decimal. You add the digits in the corresponding positions, starting from the rightmost digit. If the result is greater than 1, you carry the 1 to the next column. For example:

1101 (13 in decimal)

1. +1011 (11 in decimal)
10100 (20 in decimal)

Subtraction: Subtraction in binary works the same as subtraction in decimal. You subtract the digits in the corresponding positions, starting from the rightmost digit. If the result is negative, you borrow 1 from the next column. For example:

1101 (13 in decimal)

2. -1011 (11 in decimal)
0010 (2 in decimal)

Multiplication: Multiplication in binary works the same as multiplication in decimal. You multiply the first number by each digit of the second number, shifting the result left for each digit. For example:

1101 (13 in decimal)

3. x1011 (11 in decimal)
1110111 (143 in decimal)

4. **Division:** A division in binary works the same as a division in decimal. You divide the first number by the second number using the long division method, shifting the result right for each digit.

Bitwise Operations: Several bitwise operations can be performed on binary numbers, such as AND, OR, XOR, and NOT operations. Bitwise operations are used to manipulate the individual bits of a binary number. For example:

1101 (13 in decimal)

5. AND 1011 (11 in decimal)
and 1001 (9 in decimal)

It's important to note that while these operations are similar to their decimal counterparts, they are not the same because the base of the number system is different. It's also worth noting that, with the use of computers, these operations are usually done automatically by the computer, and you as a user don't need to compute them manually.

The requirements: Search the Following

- ERROR AND ERROR CONNECTION

Errors in computers can refer to various issues that can occur during the operation of a computer system. Some common types of errors include. An error in a computer can refer to a variety of issues that can occur during the operation of a computer system. Some common types of errors include:

1. **Hardware errors:** These are errors that occur due to a problem with the physical components of the computer, such as a malfunctioning hardware component or a faulty connection. Examples include a malfunctioning hard drive, a faulty memory module, or a malfunctioning power supply.
2. **Software errors:** These are errors that occur due to a problem with the computer's software, such as a malfunctioning program or a software bug. Examples include a crash in an operating system, an error message in a program, or a failure to install a software update.
3. **Input/output errors:** These are errors that occur when data is transmitted or received by the computer. Examples include a failure to read or write data to a disk, a failure to transmit data over a network, or a failure to read data from a keyboard or mouse.
4. **Logic errors:** These errors occur due to a problem with the logic or design of a computer program. They may not produce an error message, but the program may not produce the correct results.
5. **Human errors:** These are errors that occur due to the actions of the user, such as accidentally deleting a file, or mistakenly changing a setting.

These errors can be caused by a variety of factors and can have a wide range of impacts on the operation of the computer.

The error-detecting codes and their types

Error-detecting codes are mathematical algorithms and techniques used to detect errors in digital data transmission or storage. These codes are used to detect errors that may occur during the transmission or storage of data, and to take appropriate action to correct them. In the modern world, error-detecting codes play a crucial role in ensuring the integrity and reliability of digital communication and data storage systems.

There are several types of error-detecting codes, including:

1. **Parity codes:** These codes add an extra bit, called the parity bit, to each block of data, to indicate whether the number of 1s in the data is even or odd. This allows the receiver to detect errors by comparing the parity bit to the number of 1s in the received data.
2. **Checksum codes:** These codes add a fixed number of bits, called the checksum, to each block of data, based on the value of the data bits. The receiver can detect errors by recalculating the checksum and comparing it to the received value.
3. **Cyclic redundancy check (CRC) codes:** These codes use a mathematical algorithm to generate a fixed-length code, called the CRC, based on the data. The receiver can detect errors by recalculating the CRC and comparing it to the received value.
4. **Hash codes:** These codes use a mathematical function to generate a fixed-length code, called the hash, based on the data. The receiver can detect errors by recalculating the hash and comparing it to the received value.
5. **Error-correcting codes:** These codes can detect and correct errors, rather than just detecting them. The most common example is the Hamming code and

Reed-Solomon codes; these codes add extra bits to the data to allow the receiver to correct errors.

These codes are widely used in various fields such as computer science, network security, telecommunications, and storage systems. With the increasing reliance on digital communications and online transactions, the importance of error-detecting codes is likely to continue to grow in the future.

The Parity Check

Parity check is a method of detecting errors in digital data transmission or storage by adding an extra bit, called the parity bit, to each block of data. The value of the parity bit is chosen so that the number of 1s in the data, including the parity bit, is either even or odd.

There are two types of parity checks:

1. Even parity: In this method, the value of the parity bit is chosen so that the number of 1s in the data, including the parity bit, is always even. For example, if the data is "0101", the parity bit would be set to 1, making the total number of 1s in the data an even number (2).
2. Odd parity: In this method, the value of the parity bit is chosen so that the number of 1s in the data, including the parity bit, is always odd. For example, if the data is "0101", the parity bit would be set to 0, making the total number of 1s in the data an odd number (3).

When the receiver receives the data, it can calculate the parity of the received data and compare it to the expected parity. If the calculated parity does not match the expected parity, the receiver can conclude that an error has occurred in the transmission.

Parity check is a simple and efficient method of detecting errors, however, it has a low error detection capability, and it can detect only single-bit errors. It's mainly used in simple systems where the cost and complexity of more advanced error-detection methods are not justified.

Lesson 6

Linear Programming

Linear programming is a method of mathematical optimization that is used to find the best solution to a problem that can be expressed as a linear function. It is a technique used to optimize a linear objective function, subject to constraints represented by linear equations or inequalities. Linear programming is used in a wide range of fields, including business, economics, and operations research, to make decisions about the allocation of limited resources.

In the modern world, linear programming is used in various fields such as finance, manufacturing, transportation, and logistics. It is used to solve problems such as:

1. Production scheduling: Linear programming can be used to optimize the production schedule for a manufacturing facility, to minimize costs and maximize profits.
2. Portfolio optimization: Linear programming can be used to optimize the mix of investments in a portfolio, to maximize returns and minimize risk.
3. Resource allocation: Linear programming can be used to optimize the allocation of resources, such as labor, materials, and equipment, to achieve specific goals.
4. Traffic routing: Linear programming can be used to optimize the routing of vehicles and other transportation assets, to minimize costs and maximize efficiency.
5. Energy planning: Linear programming can be used to optimize the use of energy resources, minimize costs and maximize sustainability.

In summary, linear programming is a powerful mathematical tool that helps decision-makers to optimize the use of limited resources, it helps to achieve the best outcome under certain constraints. It's a widely used technique in many areas of the modern world, from business and finance to transportation and logistics.

Examples:

- Supermarket chains use linear programming to determine which warehouses should ship, which product, and where to store. Many businesses, industries, and government agencies use linear programming successfully. Businesses use it to determine the best way to manage personnel and workloads in a specific job.

The Linear Inequalities or Inequality tells us about the relative size of two values. We call things like these inequalities (because they are not “equal”). The symbol “<” is read as “is less than” and “>” is read as “is greater than”. The symbols like “≤” and “≥” read as “is less than or equal to” and “is greater than or equal to” respectively, are also used.

The **Linear Inequalities or Inequality** tells us about the relative size of two values. We call things like this inequalities (because they are not “equal”). The symbol “<” is read as “is less than” and “>” is read as “is greater than”. The symbols like “≤” and “≥” read as “is less than or equal to” and “is greater than or equal to” respectively, are also used.

There are Three Properties of Inequalities

The following are the properties of inequalities that you can use in dealing with problems in inequalities:

1. Addition Property of Inequality (API)
For any real numbers a , b , and c ;
If $a < b$, then $a + c < b + c$; and if $a > b$, then $a + c > b + c$.
2. Multiplication Property of Inequality (MPI)
Let a , b , c be real numbers.
If $a > b$ then $ac > bc$, for every positive number c .
If $a > b$, then $ac < bc$, for every negative number c .
If $a < b$, then $ac < bc$, for every positive number c .
If $a < b$, then $ac > bc$, for every negative number c .

3. Trichotomy Property

If $x = 0$, then x is not positive nor negative.

If $x > 0$, then $x \neq 0$ and x is positive.

If $x < 0$, then $x \neq 0$ and x is negative.

4. Transitive Property of Inequality

Let a , b , and c be real numbers.

Let $a < b$ and $b < c$, then $a < c$.

Example 1: Solve

$$3x + 2 < 14.$$

$$3x + 2 < 14 \quad \text{Given}$$

$$3x + 2 + (-2) < 14 + (-2) \quad \text{Addition Property of Inequality}$$

$$3x < 12 \quad \text{Inverse element}$$

$$\frac{1}{3}(3x) < 12 \left(\frac{1}{3}\right) \quad \text{Multiplication Property of Inequality}$$

$$x < 4$$

Geometry of Linear Programming or Linear optimization of Linear programming is a method of mathematical optimization that is used to find the best solution to a problem that can be expressed as a linear function. It is a sub-field of the broader optimization field called convex optimization. The main objective of linear programming is to maximize or minimize the numerical value. It consists of linear functions which are subjected to constraints in the form of linear equations or inequalities. The technique is used to optimize the use of limited resources and it's widely used in various fields such as business, economics, operations research, finance, manufacturing, transportation, and logistics. Linear programming is considered an important technique that is used to find the optimum resource utilization. The term "linear programming" consists of two words "linear" and "programming." "Linear" defines the relationship between multiple variables with degree one, while "programming" defines the process of selecting the best solution from various alternatives.

Hyper planes and Half spaces

Definition 1: Let a be nonzero in \mathbb{R}^n and let b be a scalar.

a. The set $\{x \in \mathbb{R}^n \mid a'x = b\}$ is called a hyperplane.

b. The set $\{x \in \mathbb{R}^n \mid a'x > b\}$ is called a half-space.

A **hyperplane** is an equation whereas a **half-space** is an inequality. For example, $a_1x_1 + a_2x_2 = b$ is a linear equation in two-dimensional space with coordinates x_1 and x_2 . This is also known as a linear equation. So a hyperplane in 2-dimensional space is a line. Similarly, in 3-dimensions $a_1x_1 + a_2x_2 + a_3x_3 = b$ is a linear equation and it is known as a plane. In more dimensions than 3, the geometric object constructed by a linear equation is called a **hyperplane**.

Half-space is similar to a hyperplane, but it covers the area on one side of a hyperplane. In the above picture, the side red dots are in a half-space, and the side green dots are in another half-space and they are separated by a hyperplane. In other words, a hyperplane is the boundary of a corresponding half-space.

Polyhedron (e.britanica) is a three-dimensional object composed of a finite number of polygonal surfaces (faces) as a boundary between the interior and exterior of a solid. In general, polyhedrons are named according to the number of faces. A tetrahedron has four faces, a pentahedron five, and so on; a cube is a six-sided regular polyhedron (hexahedron)

whose faces are squares. The faces meet at line segments called **edges**, which meet at points called **vertices**.

Convex Sets

Definition 3: A set $S \subseteq \mathbb{R}^n$ is convex if for any $x, y \in S$, and any $\lambda x + (1 - \lambda)y \in S$.

A convex set is a set of points such that, given any two points A, B in that set, the line AB joining them lies entirely within that set. That is why the left plot is convex and the right plot is non-convex.

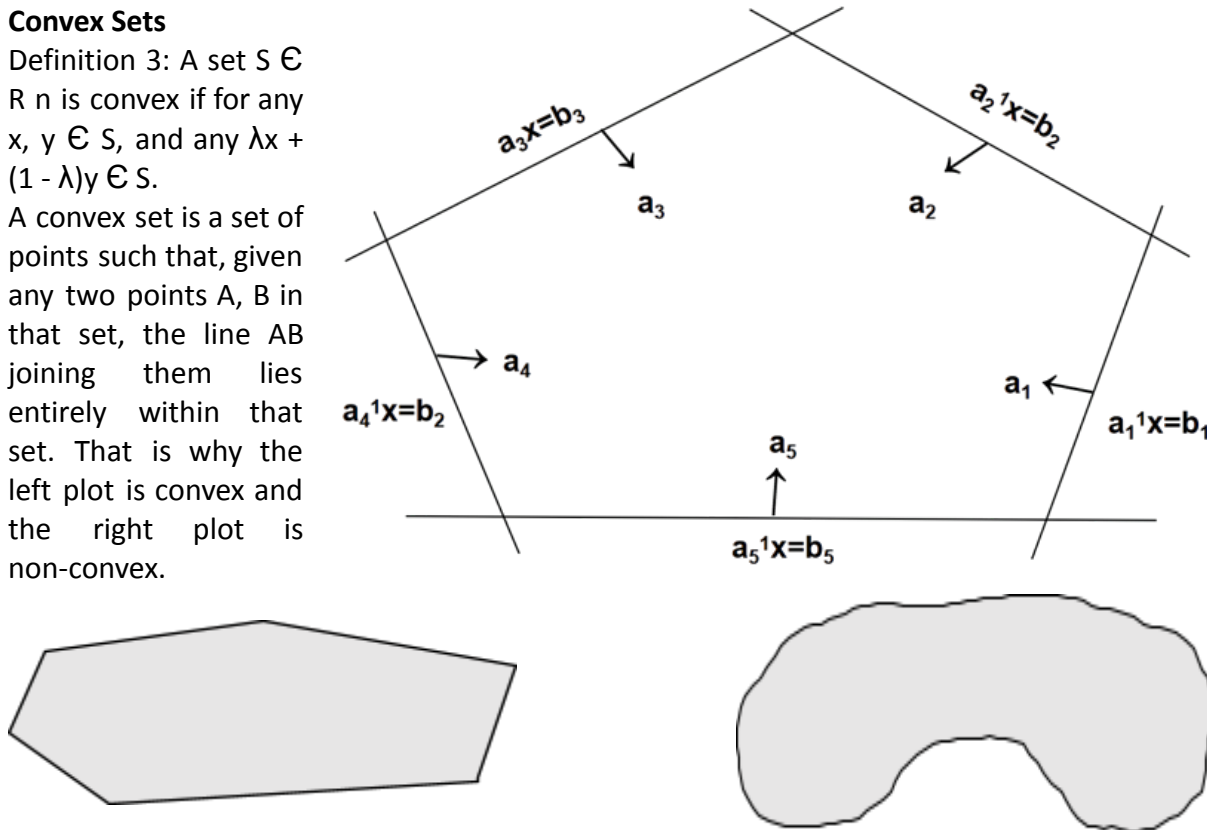


Figure 1: Examples of a convex set (a) and a non-convex set (b)

A linear programming problem or LP problem (Waner, 2016) in two unknown x and y is one in which we are to find the maximum or minimum value of a linear expression $ax + by$ called the objective function, subject to several linear constraints of the form $cx + dy < e$ or $cx + dy > e$. The solution set of the collection of constraints is called the feasible region of the LP problem. The largest or smallest value of the objective function is called the optimal value, and a pair of values of x and y that gives the optimal value constitutes an optimal solution.

Example 1: Solve the following linear programming problem using the graphical method.

Minimize: $Z = 5x + 4y$

$$4x + y \geq 40$$

$$2x + 3y \geq 90$$

$$x, y \geq 0$$

Solution: Using the constraints we get the equations of the lines as $4x + y = 40$ and $2x + 3y = 90$. $4x + y = 40$ passes through $(0, 40)$ and $(10, 0)$. Any point lying on or above this line satisfies $4x + y \geq 40$. $2x + 3y = 90$ passes through $(0, 30)$ and $(45, 0)$. Any point lying on or above this line satisfies $2x + 3y \geq 90$.

Fundamental theorem of linear programming

The following are the fundamental theorems (Podmanik, 2018):

1. If a solution exists to a bounded linear programming problem, then it occurs at one of the corner points.

2. If a feasible region is unbounded, then a maximum value for the objective function does not exist.
3. If a feasible region is unbounded, and the objective function has only positive coefficients, then a minimum value exists.

Solving a Linear Programming Problem Graphically is the importance of linear programming models in various industries, many types of algorithms have been developed over the years to solve them. Some famous mentions include the Simplex method, the Hungarian approach, and others. Here we are going to concentrate on one of the most basic methods to handle a linear programming problem i.e. the graphical method. In principle, this method works for almost all different types of problems but gets more and more difficult to solve when the number of decision variables and the constraints increases. Therefore, we'll illustrate it in a simple case i.e. for two variables only. So let's get started with the graphical method.

1. Define the variables to be optimized. The question asked is a good indicator as to what these will be.
2. Write the objective function in words, then convert it to a mathematical equation.
3. Write the constraints in words, then convert them to mathematical inequalities.
4. Graph the constraints as equations.
5. Shade feasible regions by taking into account the inequality sign and its direction. If
 - a) A vertical line
 - <, then shade to the left
 - >, then shade to the right
 - b) A horizontal line
 - <, then shade below
 - >, then shade above
 - c) A line with a non-zero, defined slope
 - <, a shade below
 - >, a shade above
6. Identify the corner points by solving systems of linear equations whose intersection represents a corner point.
7. Test all corner points in the objective function. The "winning" point is the point that optimizes the objective function(biggest if maximizing, smallest if minimizing)

Solving the Linear Programming Problem using Simplex Method

Example 1: Consider the following standard maximum-type linear programming problem.

Maximize	$P = 3x + 4y$	(objective function)
subject to	$x + 3y < 30$	first constraint
	$2x + y < 20$	second constraint
	$x > 0; y > 0$	

Using the Simplex Method, the steps are as follows:

Step 1. Insert Slack Variables

Insert a slack variable into each of the structural constraints.

The result is the system of slack equations.

$$x + 3y + s_1 = 30 \text{ (first slack equation)} \quad 2x + y + s_2 = 20 \text{ (second slack equation)}$$

Step 2. Rewrite the Objective Function

Rewrite the objective function to match the format of the slack of equations, and add the corresponding equation to the bottom of the slack of equations;

$$x + 3y + s_1 = 30, \quad 2x + y + s_2 = 20, \quad -3x - 4y + P = 0.$$

Step 3. Write the Initial Simplex Tableau

The augmented matrix is called the initial simplex tableau. Each number in the bottom row, to the left of the vertical bar, is called an indicator.

x	y	s1	s2	P	Constant
1	3	1	0	0	30
2	1	0	1	0	20
-3	-4	0	0	1	0

Step 4. Find the Pivot Element.

The most negative indicator in the last row of the tableau determines the pivot column. We apply the smallest quotient rule for that column. Find the pivot element of our initial simplex tableau below.

x	y	s1	s2	P			
R1	1	3	1	0	0	30	
R2	2	1	0	1	0	20	
R3	-3	-4	0	0	1	0	

The pivot element is the most negative indicator in the third row. In this example, the second column is the pivot column with -4 as the most negative indicator.

Step 5. Perform the Pivot Operation.

Perform the pivot operation to find the pivot element. After the pivot operation has been completed, write down the basic feasible solution.

x	y	s1	s2	P	Constant	Ratios
1	3	1	0	0	30	(30/3 = 10)
2	1	0	1	0	20	(20/1 = 20)
-3	-4	0	0	1	0	

Step 6. Check if there are still negative values in the third row

If a negative indicator is still present, repeat steps 4 and 5. If no negative indicators are present, the maximum of the objective function has been reached.

x	y	s1	s2	P	
1/3	1	1/3	0	0	10 {10/(1/3)} = 300
5/3	0	-1/3	1	0	10 {10/(5/3)} = 60 smaller
-5/3	0	4/3	0	1	40

2 Properties of Linear Programming Problems:

They deal with maximizing or minimizing some quantities.

There are restrictions or constraints that limit the degree to which the objective can be pursued.

Model Formulation:

A **linear programming model** consists of certain common components including decision variables, an objective function, and model constraints, which consist of decision variables and parameters.

4 Components of a Model

1. **Objective** - linear programming algorithms require that a single goal or objective be specified. The objective function is a linear mathematical relationship that describes the objective of the firm in terms of the decision variables.
2. **Decision Variables**- are mathematical symbols that represent levels of activity by the firm. These represent choices available to the decision maker in terms of amounts of either inputs or outputs
3. **Constraints**- are also linear relationships of the decision variables; they represent the restrictions placed on the firm by the operating environment. The restrictions can be in the form of limited resources or restrictive guidelines.
4. **Parameters**- numerical values that are included in objective functions and constraints.

Assumptions on Linear Programming Models:

- **Linearity**- the impact of decision variables is linear in constraints and the objective function.
- **Divisibility**- non-integer values of decision variables are acceptable.
- **Certainty**- values of parameters are known and constant.
- **Non-negativity**- negative value of decision variables is unacceptable.

Formulating a Linear Programming Model:

The information about the problem should be gathered but naturally, it is important to obtain valid information on what constraints are appropriate. Once all information has been obtained, you are now ready to assemble a model.

1st approach: (this follows a logical sequence)

1. Identify the decision variables.
2. Write out the objective function.
3. Formulate each of the constraints

2nd approach: (Let the problem guide you in formulating a linear programming model)

Example: If the first words of a problem are: "A manager wants to minimize profit" this would lead to an objective function: Maximize Profit. But if the words are: "A department has 1,000 pounds of raw material available to prepare its products" this would lead to raw material $< 1,000$ pounds.

Maximization Problem

Example: An appliance manufacturer produces two models of electric fans: a stand fan and a desk fan. Both models require fabrication and assembly work. Each stand fan uses 4 hours of fabrication and 2 hours of assembly work. While each desk fan uses 2 hours of fabrication and 6 hours of assembly work. There are 600 fabrication hours available per week and 480 hours of assembly work. Each stand fan contributes P400 to profits and each desk fan contributes P300 to profits. How many stand fans and desk fans must the manufacturer produce to maximize the profit?

Solution:**Decision Variables:**

Let x = several stand fans to produce

y = the number of desk fans to produce

By tabulating first, the information about the problem, we may come out with this table:

Product	Fabrication time	Assembly Work time	Profit
x	4 hours	2 hours	P400
y	2 hours	6 hours	P300
Available time	600 hours	480 hours	

Then the problem could be formulated as follows:

LP Model:

Maximize Profit = $400x + 300y$

subject to

fabrication: $4x + 2y < 600$

assemblies: $2x + 6y < 480$

non-negativity: $x > 0$

$y > 0$

GRAPHICAL LINEAR PROGRAMMING

- Involves plotting the constraints on a plane and identifying an area that will satisfy all the given constraints. This area is a feasible solution area that contains the combination of values for the decision variables that simultaneously satisfy all the restrictions in a linear program.

Steps:

Assign decision variables for the given products.

1. Set up the objective function and the constraints in a mathematical format.
2. Plot the constraints on the xy -plane after converting the inequality sign with an equal sign.
3. Identify the feasible solution area.
4. Substitute the coordinates at the points of intersection of the feasible solution area to the objective function.
5. Formulate your decision by choosing the order that gives the highest value for maximization and the lowest value if it is minimization.

Plotting the Constraints on xy -plane:

Plot the fabrication time constraints $4x + 2y < 600$ and the assembly time constraint $2x + 6y < 480$.

Graph: (task #1)

Substituting the coordinate of the vertices of the feasible region to the objective function;

Vertex	Objective Function $400x + 300y$	Total Profit
A (0, 80)	$400(0) + 300(80)$	24,000
B (132, 36)	$400(132) + 300(36)$	63,600
C (150, 0)	$400(150) + 300(0)$	60,000

Formulating the Decision:

Therefore, vertex B (132,36) will give the highest profit of P63,600. We decide to choose the one that will give the highest value since the objective is to maximize profit. In this case, the manufacturer can produce 132 units of stand fans and 36 units to maximize profit.

Minimization Problem

Example: A dietitian has learned from a nutrition book that his family needs at least 300 grams of protein and at least 60 milligrams of iron per day for sound health. These nutrients can be obtained from meat and vegetable products. Each pound of meat costs an average of P60 and contains an average of 150 grams of protein and 15 grams of iron while each pound of vegetable costs P15 and has 10 grams of protein and 5 grams of iron. He wants to determine the quantities of food that meet the nutritional requirements at the least cost.

Solution:**Decision Variables:**

Let x = several pounds of meat

y = several pounds of vegetable

LP Model:

Minimize Cost= $60x + 15y$

subject to

protein: $150x + 10y > 300$

iron: $15x + 5y > 60$

non-negativity: $x > 0$

$y > 0$

Plotting the constraints on the xy-plane:**Graph: (task #2)**

Substituting the coordinate of the vertices of the feasible region to the objective function;

Vertex	Objective Function $60x + 15y$	Total Cost
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A (0,30)	$60(0) + 15(30)$	P450
B (1.5,7.5)	$60(1.5) + 15(7.5)$	P202.5
C (4,0)	$60(4) + 15(0)$	P240

Decision:

Therefore the quantities of food that meet the nutritional requirements at least cost are 1.5 pounds of meat and 7.5 pounds of vegetables.

Vertex	Objective Function $60x + 15y$	Total Cost
A (0,30)	$60(0) + 15(30)$	P450
B (1.5,7.5)	$60(1.5) + 15(7.5)$	P202.5
C (4,0)	$60(4) + 15(0)$	P240

Decision:

Therefore the quantities of food that meet the nutritional requirements at least cost are 1.5 pounds of meat and 7.5 pounds of vegetables.

Lesson 7

Mathematics of Finance

The mathematics of finance involves the use of mathematical techniques to model and analyze financial systems and markets. It is a broad field that encompasses various areas such as portfolio theory, risk management, financial derivatives, and asset pricing. Some of the key concepts in the mathematics of finance include:

1. Time value of money: This concept states that money has a different value at different points in time, taking into account factors such as inflation and interest rates.
2. Interest: Interest is the cost of borrowing money, and it is calculated as a percentage of the principal amount.
3. Compound interest: This is interest that is calculated not only on the original principal but also on the accumulated interest from previous periods.
4. Annuities: An annuity is a series of payments made at regular intervals, such as monthly or annually.

5. Bond pricing: Bonds are debt securities issued by companies or governments, and the price of a bond is determined by the interest rate, the term of the bond, and the creditworthiness of the issuer.
6. Options: An option is a contract that gives the buyer the right, but not the obligation, to buy or sell an underlying asset at a specific price and date.
7. Futures: A future is a contract to buy or sell an underlying asset at a specific price and date in the future.
8. Portfolio optimization: The mathematics of finance also includes the study of how to optimize the mix of investments in a portfolio, to maximize returns and minimize risk.

In summary, The mathematics of finance is a broad field that encompasses various areas such as portfolio theory, risk management, financial derivatives, and asset pricing. It's important to understand and use mathematical concepts and techniques to model and analyze financial systems and markets.

Simple and Compound Interest

- Simple interest and compound interest are two ways of calculating the interest earned on a principal amount of money over a certain period.

Simple interest is calculated as the product of the principal, the interest rate, and the time.

The formula for simple interest is: $\text{Simple Interest} = \text{Principal} \times \text{Interest Rate} \times \text{period}$

- For example, if you invest \$1000 at a simple interest rate of 5% per year for 2 years, the simple interest earned would be \$100 ($\$1000 \times 0.05 \times 2$).

Compound interest, on the other hand, is calculated on the principal amount and the accumulated interest from previous periods. The formula for compound interest is:

$\text{Compound Interest} = \text{Principal} \times (1 + \text{Interest Rate})^{\text{Time Period}} - \text{Principal}$

- For the same example, if you invest \$1000 at a compound interest rate of 5% per year for 2 years, the compound interest earned would be \$102.50. This is because, in the first year, the interest earned would be \$50 ($\1000×0.05) and in the second year, the interest would be calculated on the new balance of \$1050 ($\$1000 + \50) resulting in \$52.50 ($\1050×0.05).

In summary, Simple interest is calculated only on the principal amount while Compound interest is calculated on the principal and the accumulated interest from previous periods. Compound interest is generally higher than simple interest over longer periods due to the compounding effect which makes the interest grow exponentially.

Simple Interest-Future Value

- The future value (FV) of an investment with simple interest is the amount of money that the investment will be worth at a certain point in the future, assuming the interest is not reinvested. The formula for calculating the future value of a simple interest investment is $\text{FV} = \text{Principal} \times (1 + \text{Interest Rate} \times \text{period})$

For example, if you invest \$1000 at a simple interest rate of 5% per year for 2 years, the future value would be \$1100 ($\$1000 \times (1 + 0.05 \times 2)$).

It is important to note that in simple interest, the interest earned is not added to the principal and the interest is not compounded. Therefore, the future value of an investment with simple interest is always equal to the principal plus the interest earned. In summary, The future value of an investment with simple interest is the amount of money that the investment will be worth at a certain point in the future, assuming the interest is not reinvested. It is calculated by multiplying the principal by $(1 + \text{Interest Rate} \times \text{Time Period})$.

Simple Interest-Present Value

- The present value (PV) of an investment with simple interest is the current value of an investment, taking into account the future value and the time value of money. It represents how much money you would need to invest today to receive a certain future value, assuming a simple interest rate.

The formula for calculating the present value of a simple interest investment is:

- $PV = FV / (1 + \text{Interest Rate} \times \text{period})$. Where FV is the future value of the investment and Interest Rate and Period are the interest rate and period for the investment.
- For example, if you want to invest \$1000 in 2 years at a simple interest rate of 5% per year, the present value would be \$952.38 ($\$1000 / (1 + 0.05 \times 2)$).

It is important to note that in simple interest, the interest earned is not added to the principal, and the interest is not compounded. Therefore, the present value of an investment with simple interest is always equal to the future value divided by $(1 + \text{Interest Rate} \times \text{period})$. In summary, The present value (PV) of an investment with simple interest is the current value of an investment, taking into account the future value and the time value of money. It's calculated by dividing the future value by $(1 + \text{Interest Rate} \times \text{period})$ where FV is the future value, Interest rate and Period are the interest rate and period for the investment.

Simple discount notes

- Simple discount notes, also known as promissory notes, are financial instruments that represent a loan between two parties. The lender, or the person issuing the note, promises to pay a certain amount of money to the borrower at a specific date in the future. The borrower, in turn, promises to pay the lender a discounted amount of money, which is known as the face value of the note, at the time of maturity.

The formula for calculating the face value of a simple discount note is: $\text{Face Value} = \text{Maturity Value} / (1 + \text{Interest Rate} \times \text{Time Period})$

- Where Maturity Value is the amount to be paid at the time of maturity, Interest Rate is the interest rate for the note and Period is the number of years for which the note is issued.
- For example, if a borrower wants to borrow \$1000 in 2 years at a simple interest rate of 5% per year, the face value of the note would be \$952.38 ($\$1000 / (1 + 0.05 \times 2)$)

It is important to note that the interest earned on a simple discount note is the difference between the face value and the maturity value and is calculated using the simple interest formula. In summary, Simple discount notes, also known as promissory notes, are financial instruments that represent a loan between two parties. The lender promises to pay a certain

amount of money to the borrower at a specific date in the future, and the borrower promises to pay the lender a discounted amount of money, which is known as the face value of the note, at the time of maturity. The face value of the note is calculated by dividing the maturity value by $(1 + \text{Interest Rate} \times \text{Time Period})$.

Compound interest

is the interest on a loan or deposit calculated based on both the initial principal and the accumulated interest from previous periods. The interest earned in each period is added to the principal and the interest is calculated on the new, larger principal.

The formula for calculating the compound interest is:

$$A = P(1 + r/n)^{nt}$$

Where:

- A is the future value of the investment (principal + interest)
- P is the principal or initial investment
- r is the annual interest rate
- n is the number of times the interest is compounded per year
- t is the number of years for the investment
- For example, if you invest \$1000 at an annual interest rate of 5% compounded annually for 2 years, the future value of the investment would be \$1102.50

$$A = \$1000 \times (1 + 0.05)^2 = \$1102.50$$

- Another example if you deposit \$1000 in a savings account that pays an annual interest rate of 5% compounded semi-annually (twice per year) for 2 years, the future value of your deposit would be \$1105.06

$$A = \$1000 \times (1 + (0.05/2))^{(2 \times 2)} = \$1105.06$$

It is important to note that the compound interest formula assumes that the interest is compounded continuously, which is not always the case in real-world scenarios. However, the formula can be adjusted for other compounding frequencies. In summary, Compound Interest is the interest on a loan or deposit calculated based on both the initial principal and the accumulated interest from previous periods. The interest earned in each period is added to the principal and the interest is calculated on the new, larger principal. The formula for calculating the compound interest is $A = P(1 + r/n)^{nt}$ where A is the future value of the investment, P is the principal or initial investment, r is the annual interest rate, n is the number of times the interest is compounded per year, t is the number of years for the investment.

Compound Interest-Effective Rate

- The effective interest rate, also known as the effective annual rate (EAR), is the true annual cost of borrowing or the true annual yield of an investment, taking into account the effects of compounding. It is a standard way of expressing the interest rate for investments with compound interest.

The formula for calculating the effective interest rate for a compound interest investment is:

- $\text{EAR} = (1 + \text{Interest Rate} / \text{Number of Compounding Periods})^{\text{Number of Compounding Periods}} - 1$

Where Interest Rate is the stated annual interest rate and the Number of Compounding Periods is the number of times per year that interest is added to the principal.

- For example, if an investment has an annual interest rate of 5% and compounds annually, the effective interest rate would be 5%. However, if the same investment compounds quarterly, the effective interest rate would be 5.0625% ($(1 + 0.05/4)^4 - 1$).

It's important to note that the effective interest rate takes into account the compounding effect on the interest rate, so the effective rate is always higher than the nominal interest rate. In summary, the Effective Interest Rate (EAR), also known as the effective annual rate, is the true annual cost of borrowing or the true annual yield of an investment, taking into account the effects of compounding. The formula for calculating the effective interest rate for a compound interest investment is $(1 + \text{Interest Rate} / \text{Number of Compounding Periods})^{\text{Number of Compounding Periods}} - 1$, where Interest Rate is the stated annual interest rate, and Number of Compounding Periods is the number of times per year that interest is added to the principal.

Compound Interest-Present Value

- The present value (PV) of an investment with compound interest is the current value of an investment, taking into account the future value, interest rate, and time value of money. It represents how much money is invested today to receive a certain future value, assuming compound interest.

The formula for calculating the present value of a compound interest investment is:

$$\text{PV} = \text{FV} / (1 + r)^n$$

- Where FV is the future value of the investment, r is the interest rate and n is the number of compounding periods.
- For example, if you want to invest \$1000 in 2 years at a compound interest rate of 5% per year, the present value would be \$903.09 ($\$1000 / (1 + 0.05)^2$)

It is important to note that in compound interest, the interest earned is added to the principal and the interest is compounded. Therefore, the present value of an investment with compound interest is always equal to the future value divided by $(1 + r)^n$. In summary, The present value of an investment with compound interest is the current value of an investment, taking into account the future value, interest rate, and time value of money. It represents how much money you would need to invest today to receive a certain future value, assuming compound interest. It is calculated by dividing the future value by $(1 + r)^n$, where FV is the future value, r is the interest rate and n is the number of compounding periods.

Compound Interest using Logs

- The formula for compound interest can be expressed using logarithms, specifically natural logarithms (ln). This method is useful for solving compound interest problems that involve large numbers, such as those with many compounding periods.

- The formula for calculating the future value of an investment with compound interest using logarithms is: $FV = PV * e^{(r*t)}$. Where PV is the present value of the investment, r is the annual interest rate (expressed as a decimal), t is the number of years the investment is held, and e is the base of the natural logarithms (approximately 2.718).
- For example, if you invest \$1000 at an annual interest rate of 5% compounded annually for 2 years, the future value of the investment would be \$1276.28 ($\$1000 \times e^{(0.05*2)}$).

It is important to note that the $e^{(r*t)}$ term in the formula represents the compound interest factor, which increases exponentially with time. This is why compound interest has a much greater effect on the growth of an investment than simple interest. In summary, The formula for compound interest can be expressed using logarithms, specifically natural logarithms (ln) to make it more efficient to calculate the future value of an investment. The formula is $PV * e^{(r*t)}$ where PV is the present value of the investment, r is the annual interest rate (expressed as a decimal), t is the number of years the investment is held, and e is the base of the natural logarithms (approximately 2.718).

The followings are some of the transactions where a type of interest is applied;

1. Savings accounts: Interest is applied to money deposited into a savings account and is typically compounded regularly, such as monthly or annually.
2. Certificates of deposit (CDs): Interest is applied to money deposited into a CD and is typically compounded regularly, such as monthly or annually. CDs have a fixed term, and a higher interest rate is typically offered for longer terms.
3. Bonds: Interest is paid to bondholders, typically on a semi-annual basis. The interest rate is known as the coupon rate, and the interest is compounded semi-annually.
4. Mortgages: Interest is applied to the principal amount of a mortgage loan. The interest rate can be fixed or adjustable, and it is typically compounded every month.
5. Credit cards: Interest is applied to the outstanding balance on a credit card account. The interest rate is typically higher than other types of loans and it is usually compounded daily.
6. Auto loans: Interest is applied to the principal amount of an auto loan. The interest rate can be fixed or adjustable, and it is typically compounded every month.
7. Personal loans: Interest is applied to the principal amount of a personal loan. The interest rate can be fixed or adjustable, and it is typically compounded every month.
8. Investment accounts: Interest is applied to money invested in a variety of accounts, such as mutual funds or exchange-traded funds (ETFs). The interest is typically compounded regularly, such as daily or annually.

In summary, There are many transactions where a type of interest is applied, such as savings accounts, Certificates of deposit, Bonds, Mortgages, Credit cards, Auto loans, Personal loans, and Investment accounts. The interest rate, compounding period, and frequency of payment vary depending on the type of account or loan.

As an I.T. student, how can we apply the subject and its lessons: Mathematics in the Modern World in the field of Information Technology?

As an IT student, mathematics plays a crucial role in various aspects of the field. Some ways in which mathematics can be applied In the field of Information Technology, mathematical concepts such as algorithms, data structures, number theory, probability theory, linear algebra, calculus, statistics, optimization, graph theory, logic, and set theory are essential in designing and implementing efficient and effective solutions to problems in computer science. These mathematical concepts are used in various areas such as cryptography for secure communication systems and data encryption, computer graphics for creating and manipulating images, animations, and virtual environments, machine learning for building and training algorithms, computer networks for routing and scheduling, database systems for query optimization and data integrity and Artificial Intelligence for building and training neural networks and improving the performance of AI systems.

In summary, as an IT student, mathematics is a fundamental subject that plays a critical role in various aspects of the field, such as algorithms and data structures, cryptography, computer graphics, machine learning, computer networks, database systems, and Artificial Intelligence. Understanding and applying mathematical concepts can help IT students to design and implement efficient and effective solutions to problems in computer science and technology.

